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Plane Symmetric Anisotropic String Cosmological Model with Electromagnetic Field in General Relativity

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Abstract: In the present paper, plane symmetric anisotropic string cosmological model in the presence of electromagnetic field is investigated. The metric potentials are functions of x and t both. We have assumed that F_{12} is the only non-vanishing component of electromagnetic field tensor F_{ij}. To get the deterministic solution, it has been assumed that the expansion (θ) in the model is proportional to the eigen value $\sigma^1{}_1$ of the shear tensor $\sigma^i{}_j$. It is observed that the model has a Barrel type singularity. Some physical and geometrical properties of the model are also discussed.

Keywords: Cosmic string, Electromagnetic field, Inhomogeneous universe.

INTRODUCTION

In recent years cosmological models exhibiting plane symmetry have attracted the attention of various authors. At the present state of evolution, the universe is spherically symmetric and the matter distribution in it is isotropic and homogeneous. But in its early stages of evolution, it could have not had a smoothed out picture. So, we consider plane symmetry, which is less restrictive than spherical symmetry and provides an avenue to study inhomogeneities. The investigation of cosmic strings and their physical processes near such strings has received wide attention because it is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies. These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings by using Einstein's equations. The general treatment of strings was initiated by Letelier and Stachel[1]. The general solution of Einstein's field equations for a cloud of strings with spherical, plane and a particular case of cylindrical symmetry are obtained by Letelier2.

Letelier[3] also obtained massive string cosmological models in Bianchi type-I and Kantowski-Sachs space-times. Benerjee *et al.*[4] have investigated an axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field using a supplementary condition $\alpha = a\beta$ between metric potential where $\alpha = \alpha(t)$ and $\beta = \beta(t)$ and a is constant. Exact solutions of string cosmology for Bianchi type-Il, VI_0 ,VIII and IX space-times have been studied by Krori *et al*[5] and Wang[6]. Bali *et al.*[7-10] have investigated Bianchi type-I, III, V and type - IX string cosmological models in general relativity. The string cosmological models with a magnetic field are discussed by Chakraborty[11], Tikekar and Patel[12, 13]. String cosmological models with magnetic field in the context of space-time with G_3 symmetry are obtained by Singh and Singh[14]. Singh[15] has studied string cosmology with electromagnetic fields in Bianchi type - II, VIII and IX space-time.

Bali and Upadhaya[16] investigated LRS Bianchi type- I string dust magnetized cosmological models. Bali and Tyagi[17,18] also obtained cylindrically symmetric inhomogeneous cosmological model and stiff fluid universe with electromagnetic field in general relativity. Sharma *et al.*[19] have obtained inhomogeneous Bianchi type VI_0 string cosmological model for stiff fluid distribution. The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out Zel'dovich[20]. Also Harrison[21] has suggested that magnetic field could have a cosmological origin. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model [22]. The presence of primordial magnetic fields in the early stages of the evolution of the universe has been discussed by several authors [23-31]. Melvin [32] in his cosmological solution for dust and electromagnetic field suggested that during the evolution of the universe, the matter was in a highly ionized state and was smoothly coupled with the field, subsequently forming neutral matter as a result of universe expansion. Hence the presence of magnetic field in string dust universe is not unrealistic. Patel and Maharaj[33] investigated stationary rotating world model with magnetic field. Ram and Singh [34] obtained some new exact solution of string cosmology with and without a source free magnetic field for Bianchi type I space-time. String cosmological models with electromagnetic field have been studied by Carminati and McIntosh[35]. Lidsey, Wands and Copeland[36] have reviewed aspects of super string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Bali *et al.*[37] have investigated Bianchi type I magnetized string cosmological models. Pradhan *et al.*[38] have investigated string cosmological model in cylindrically symmetric inhomogeneous universe with electromagnetic field.

In this paper, plane symmetric anisotropic string cosmological model in the presence of electromagnetic field is investigated. The metric potentials are functions of x and t both. We have assumed that F_{12} is the only non-vanishing component of electromagnetic field tensor F_{ij}. To get the deterministic solution, it has been assumed that the expansion (θ) in the model is proportional to the eigen value $\sigma^1{}_1$ of the shear tensor $\sigma^i{}_j$. It is observed that the model has a Barrel type singularity. The physical and geometrical properties of the model are also discussed.

THE METRIC AND FIELD EQUATIONS

We consider the metric in the form

$$ds^2 = dx^2 - dt^2 + B^2 dy^2 + C^2 dz^2 \quad \dots (1)$$

Where B and C are both functions of x and t. The energy-momentum tensor for the string with electromagnetic field has the form,

$$T_i^j = \rho \, \mathrm{v_i} \, \mathrm{v^j} - \lambda x_i x^j + E_i^j \quad \dots (2)$$

With $\rho = \rho_p + \lambda$ and v_i and x_i satisfy conditions,

$$v_i v^i = -1 = -x_i x^i \quad \dots (3)$$

and

$$v^i x_i = 0 \quad \dots (4)$$

Here ρ is the rest energy density of strings with massive particles attached to them $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ is the density of tension that characterizes the strings. The unit space like vector x^i represents the string direction, i.e. the direction of anisotropy and the unit time like vector v^i describes the four velocity vector of the matter satisfying the following conditions,

$$g_{ij} v^i v^j = -1 \quad \dots (5)$$

In equation (2), E^i_j is the electromagnetic field given by Lichnerowicz [39]

$$E^i{}_j = \bar{\mu}\left[h_l h^l \left(v_i v^j + \frac{1}{2} g^i{}_j \right) - h_i h^j \right] \quad \dots (6)$$

Where $\bar{\mu}$ is the magnetic permeability and h_i is the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}} * F_{ji} v^j \qquad \ldots (7)$$

Where the dual electromagnetic field tensor $*F_{ij}$ is defined as

$$* F_{ji} = \frac{\sqrt{-g}}{2} \varepsilon_{ijkl} F^{kl} \qquad \ldots (8)$$

Here F_{ij} is the electromagnetic field tensor and ε_{ijkl} is the Levi-Civita tensor density.

The components of electromagnetic field are obtained as

$$E_1^1 = E_2^2 = E_4^4 = \frac{F_{12}{}^2}{2\bar{\mu}B^2}, \qquad E_3^3 = -\frac{F_{12}{}^2}{2\bar{\mu}B^2} \qquad \ldots (9)$$

The comoving coordinates are taken as,

$$v^i = (0,0,0,1) \qquad \ldots (10)$$

We choose the direction of string parallel to x- axis so that

$$x^i = (1,0,0,0) \qquad \ldots (11)$$

We consider the current as flowing along the z-axis so that F_{12} is the only non - vanishing component of F_{ij}. Maxwell's equations require that F_{12} is the function of x and t both and the magnetic permeability is the functions of x and t both. The semicolon represents a covariant differentiation.

$$F_{[ij;k]} = 0 \qquad \ldots (12)$$

$$\left[\frac{F^{ij}}{\bar{\mu}}\right]_{;j} = 0 \qquad \ldots (13)$$

The Einstein's field equation in the geometrized unit (c = 1, 8πG = 1)

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \qquad \ldots (14)$$

for the line-element (1) lead to the following system of equations are

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{B_1 C_1}{BC} = \lambda - \frac{F_{12}{}^2}{2\bar{\mu}B^2} \qquad \ldots (15)$$

$$\frac{C_{44}}{C}-\frac{C_{11}}{C}=-\frac{F_{12}^{\;\;2}}{2\bar{\mu}B^{2}} \qquad \ldots (16)$$

$$\frac{B_{44}}{B}-\frac{B_{11}}{B}=\frac{F_{12}^{\;\;2}}{2\bar{\mu}B^{2}} \qquad \ldots (17)$$

$$\frac{B_{4}C_{4}}{BC}-\frac{B_{11}}{B}-\frac{C_{11}}{C}-\frac{B_{1}C_{1}}{BC}=\rho-\frac{F_{12}^{\;\;2}}{2\bar{\mu}B^{2}} \qquad \ldots (18)$$

$$\frac{B_{14}}{B}+\frac{C_{14}}{C}=0 \qquad \ldots (19)$$

Where the sub-indices 1 and 4 in B ,C and elsewhere denote ordinary differentiation with respect to x and t respectively.

SOLUTION OF FIELD EQUATIONS

To find the deterministic solution of line element (1), we assume that

$B = C^n$

Where $C = f(x)\, g(t)$ … (20)

Using equations (16) and (17) we have

$$\frac{B_{44}}{B}-\frac{C_{44}}{C}+\frac{C_{11}}{C}-\frac{B_{11}}{C}=\frac{F_{12}^{\;\;2}}{\bar{\mu}B^{2}} \qquad \ldots (21)$$

and

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{C_{11}}{C}-\frac{B_{11}}{C}=0 \qquad \ldots (22)$$

From equation (22), we obtain

$$\frac{C_{44}}{C}+\frac{B_{44}}{B}=\frac{C_{11}}{C}+\frac{B_{11}}{C}=\alpha(const.) \qquad \ldots (23)$$

Which leads to

$$\frac{C_{44}}{C}+\frac{B_{44}}{B}=\alpha \qquad \ldots (24)$$

and

$$gg_{44} + rg_4^2 = sg^2 \quad \ldots (25)$$

Where $r = \frac{n(n-1)}{n+1}$, $s = \frac{\alpha}{n+1}$

Integrating equation (26), we obtain

$$g = \beta \sinh^{\frac{1}{r+1}}(bt + t_0) \quad \ldots (26)$$

Where $\beta = C_2^{1/r+1}$, $b = \sqrt{s(r+1)}$ and t_0, C_2 are constants of integration.

Again From equation (23), we have

$$ff_{11} + rh_1^2 = sf^2 \quad \ldots (27)$$

Where $r = \frac{n(n-1)}{n+1}$, $s = \frac{\alpha}{n+1}$

Integrating equation (28), we obtain

$$f = \delta \sinh^{\frac{1}{r+1}}(bx + x_0) \quad \ldots (28)$$

Where $\delta = C_4^{1/r+1}$, $b = \sqrt{s(r+1)}$ and x_0, C_4 are constants of integration.

Hence, we obtain the value of metric potential

$$B = C^n = [f(x)g(t)]^n$$
$$= P \sinh^{\frac{n}{r+1}}(bt + t_0) \sinh^{\frac{n}{r+1}}(bx + x_0) \quad \ldots (29)$$

Where $P = (\beta\delta)^n$

$$C = f(x)g(t)$$
$$= Q \sinh^{\frac{1}{r+1}}(bt + t_0) \sinh^{\frac{1}{r+1}}(bx + x_0) \quad \ldots (30)$$

Where $Q = \beta\delta$

Therefore after using suitable transformation of coordinates, metric (1) reduces to

$$ds^2 = (dX^2 - dT^2) + P^2 \sinh^{\frac{2n}{r+1}}(cX) \sinh^{\frac{2n}{r+1}}(bT) dY^2$$
$$+ Q^2 \sinh^{\frac{2}{r+1}}(bT) \sinh^{\frac{2}{r+1}}(cX) dZ \quad \ldots (31)$$

Where$bX = bx + x_0$, $bT = bt + t_0$, $Y = Py$ and $Z = Qz$

SOME PHYSICAL AND GEOMETRICAL FEATURES

The physical and geometrical properties of the model (31) are given as follows:

The magnitude of rotation ω is zero i.e. ω = 0

String Tension λ of the model is given by,

$$\lambda = \frac{b^2(1+n)}{(1+r)} + \frac{\{n(n-r)-r\}b^2}{(r+1)^2}\coth^2(bT) - \frac{b^2 n}{(r+1)^2}\coth^2(bX) + \frac{F_{12}{}^2}{\bar{\mu}P^2 \sinh^{2n/(r+1)}(bX)\sinh^{2n/(1+r)}(bT)} \quad \dots (32)$$

The energy density ρ of the model is given by,

$$\rho = -\frac{b^2(1+n)}{(1+r)} - \frac{\{n(n-r)-r\}b^2}{(r+1)^2}\coth^2(bX) + \frac{b^2 n}{(r+1)^2}\coth^2(bT) + \frac{F_{12}{}^2}{\bar{\mu}P^2 \sinh^{2n/(r+1)}(bX)\sinh^{2n/(1+r)}(bT)} \quad \dots (33)$$

The particle density ρ_p of the model is given by,

$$\rho_p = \rho - \lambda = \frac{[n-\{n(n-r)-r\}]b^2}{(r+1)^2}\coth^2(bX) - \frac{2b^2(1+n)}{(r+1)} + \frac{[n-\{n(n-r)-r\}]b^2}{(r+1)^2}\coth^2(bT) \quad \dots (34)$$

The scalar expansion θ of the model is given by,

$$\theta = \frac{b(1+n)}{1+r}\coth(bT) \quad \dots (35)$$

The Shear Scalar σ of the model is given by,

$$\sigma^2 = \frac{1}{3}\left[\frac{n^2-n+1}{(1+r)^2}\right]b^2\coth^2(bT) \quad \dots (36)$$

The proper volume V of the model is given by,

$$V^3 = PQ\sinh^{1+n/(r+1)}(bT)\sinh^{n+1/(r+1)}(bX) \quad \dots (37)$$

The deceleration parameter q of the model is given by,

$$q = -1 + \frac{3(1+r)}{(1+n)}\left[1 - \tanh^2(bT)\right] \qquad \ldots (38)$$

From equation (35) and (36) we obtain

$$\frac{\sigma}{\theta} = \sqrt{\frac{n^2 - n + 1}{3(1+n)}} = const. \qquad \ldots (39)$$

CONCLUSION

We have investigated plane symmetric anisotropic string cosmological model in the presence of electromagnetic field. The model starts with big bang at T = 0 and goes on expanding indefinitely. Since $\frac{\sigma}{\theta} \neq 0$, hence the model does not approach isotropy in general. However, if $n^2 - n + 1 = 0$ then $\frac{\sigma}{\theta} = 0$, which leads to the isotropy of the universe. When X → 0, T → 0, we observe that ρ, λ, ρ_p tend to ∞ and the proper volume V ^{3} increases as time increases. The model has a Barrel type singularity at T=0. It is observed from equation (38) that deceleration parameter $q < 0$ when $\frac{3(1+r)}{(1+n)}\left[1 - \tanh^2(bT)\right]$ <0, which represent an accelerating model of the universe. Also when $\frac{3(1+r)}{(1+n)}\left[1 - \tanh^2(bT)\right]$ = 0, the deceleration parameter $q = -1$ as in the case of de-sitter universe. In general the model represents expanding, shearing and non-rotating universe.

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REFERENCE

1. P.S. Letelier and J. Stachel. *Phys. Rev. D.*1980, **21**, 2171.
2. P.S. Letelier. *Phys. Rev. D.* 1979, **20** (6), 1294.
3. P.S. Letelier. *Phys. Rev. D.*1983, **28** (10), 2414.
4. A. Banerjee *et al.PramanaJ.phy*. 1990, **34,** 1.
5. K.D. Krori *et al*. Gen. Relativ. Gravit. 1990, **22**, 123.
6. X.X. Wang. *Chin. Phy. Lett.* 2003, **20**, 615.
7. R. Bali and Anjali. *Astrophy. Space Sci*. 2006, **302**, 20.
8. R. Bali and S. Dave. *Astrophy. Space Sci*. 2003, **288**, 399.
9. R. Bali and D. K. Singh. *Astrophy. Space Sci.* 2005, **300**, 387
10. R. Bali and A. Pradhan. *Chinese Phy. Lett*. 2007, **24**, 585.
11. S. Chakraborty. *Indian J. of Pure and Appl. Phy*. 1991, **29**, 31.

12. R. Tikekar, L.K. Patel. *Gen. Relativ. Gravit*. 1992, **24**, 397.
13. R. Tikekar and L.K. Patel. *Pramana J. Phy*. 1994, **42**, 483.
14. G.P. Singh, and T. Singh. *Gen. Relativ. Gravit*. 1999, **31**, 371.
15. G.P. Singh. *II Nuovo Cimento B.* 1995,**110**, 1463.
16. R. Bali, and R.D. Upadhaya, *Astrophy. Space Sci*. 2003, **283** (1), 97.
17. R. Bali and A. Tyagi. *Gen. Relativ. Gravit*. 1989, **21**(8), 797.
18. R. Bali, and A. Tyagi. *Astrophy. Space Sci*. 1990, **163**, 179.
19. A. Sharma, A. Tyagi & D. Chhajed. *Prespacetime J*. 2016, **7**, 615.
20. Ya. B. Zel'dovich, A.A. Ruzmainkin, and D.D. Sokoloff. *Magnetic field in Astrophys.* (Gordon and Breach New York) 1983.
21. E.R. Harrison. *Phys. Rev. Lett*. 1973, **30**, 188.
22. H.P. Robertson, A.G. Walker. *Proc. London Math. Soc*. 1936, **42**, 90.
23. C. W. Misner, K. S. Thorne. J.A. Wheeler. Gravitation W.H. Freeman, New York, 1973.
24. E. Asseo, and H. Sol. *Phys. Rep*. 1987, **6**, 148.
25. R. Pudritz and J. Silk. *Astrophys. J.* 1989, **342**, 650.
26. K.T. Kim, P. G. Tribble and P. P. Kronberg. *Astrophys. J.* 1991, **379**, 80.
27. R. Perley and G. Taylor. *Astrophys. J.* 1991, **101**, 1623.
28. P.P. Kronberg, J.J. Perry, E.L. Zukowski. *Astrophys. J.* 1991,**387**,528.
29. A.M. Wolfe, K. Lanzetta and A.L. Oren. *Astrophys. J.* 1992, **388**, 17.
30. R.R. Kulsrud, J.P. Cen, D. Ostriker and *Ryu. Astrophys. J.* 1997, **380**, 481.
31. J.D. Barrow. *Phys. Rev. D.* 1997, **55**, 7451.
32. M.A. Melvin. *Ann. New York Acad. Sci.* 1975, **262**, 253.
33. L.K. Patel and S.D. Maharaj. *Pramana J. Phys.* 1996, **47**, 1.
34. S. Ram and T.K. Singh. *Gen. Rel. Grav*. 1995, **27**, 1207.
35. J. Carminati and C.B.G. McIntosh. *J. Phys. A. Math. Gen.* 1980, **13**, 953.
36. J.E. Lidsey, D. Wands and E.J. Copeland. *Phys. Rep*. 2000, **337**, 343.
37. R. Bali, U.K. Pareek and A. Pradhan. *Chin. Phys. Lett.* 2007, **24**, 2455.
38. A. Pradhan, R. Bali and A. Rai. *Prespacetime J.* 2014, **5**, 695.
39. Lichnerowicz. W.A. Benjamin Inc New York. 1967, 93.

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