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A theoretical study of super fluidity of an atomic Fermi gas near the unitary limit and evaluation of gap function as a function of K/K_F in the unitary limit

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Abstract: Using the theoretical formalism of ***S.De Palo etal (Laser Phys. 15, 376 (2005)),*** we have theoretically studied super fluidity of an atomic Fermi gas near the unitarily limit. Our theoretically evaluated values of gap function $\Delta(k)$ as a function of $\frac{k}{k_F}$ for different values of $g\sqrt{n}/E_F$ show that the gap function has a damped oscillatory behaviour. Our theoretically evaluated results of the ratio $\frac{\Delta_0}{E_F}$ and $\frac{\mu}{E_F}$ as a function of $g\sqrt{n}/E_F$ show that $\frac{\Delta_0}{E_F}$ decreases and $\frac{\mu}{E_F}$ increases as a function of

$g\sqrt{n}/E_F$. For large value of $g\sqrt{n}/E_F$, $\frac{\Delta_0}{E_F}$ becomes constant while $\frac{\mu}{E_F}$ increases and becomes positive from negative values. These results also indicate that the resonance is along BEC side. Our theoretically evaluated results of $\frac{\Delta_0}{k_\beta T_F}$ as a function $\frac{T}{T_F}$ for different values of $g\sqrt{n}/E_F$ indicate that the values decrease as $\frac{T}{T_F}$ for all values of $g\sqrt{n}/E_F$. We observed that the narrow resonance favours occurrence of super fluidity. It is also observed that the smooth temperature behaviour is the manifestation of short ranged, bosonic character of the result. Using the theoretical formalism of ***Q. Chen [Phys. Rev. A86, 023610 (2012)],*** we have theoretically evaluated the super fluid transition temperature in a unitary atomic Fermi gas on a lattice. Our theoretically evaluated result of $\frac{T_C}{6t}$ as a function of $\frac{-U}{6t}$ for different densities show that maximum T_C occurs on the BEC side of unitary limit. We theoretically evaluated results of $\frac{T_C}{E_F}$ as a function of $\frac{-U}{6t}$ for different densities which indicate that T_C has a functional form $T_C \alpha \frac{-t^2}{U}$. Our theoretically evaluated result of $\frac{T_C}{E_F}$ as a function of $n^{\frac{1}{3}}$ show that $\frac{T_C}{E_F}$ exhibits higher order nonlinear dependence of $n^{\frac{1}{3}}$. The minimum value of T_C is on BCS region. Our theoretically evaluated results are in good agreement with the other theoretical workers.

These theoretical studies may be relevant for constructing a complete theoretical description of the crossover from BEC of composite bosons to BCS of Cooper pairs. Till today, we do not have very successful reliable theory for high T_C-superconductors. It is our firm belief that BCS-BEC crossover physics can give some valuable insight into this direction.

Keywords: Feshbach resonance, BCS-BEC crossover, unitary limit, super fluidity of atomic Fermi gas, mixtures of two hyperfine states, tightly bound bosonic dimmers, strong attractive interaction, perturbation many-body technique, Imblance Fermi gas, Fulde-Farrell-Larkin-OvehniKov (FFLO) phase, Quark-Gluon Plasma (QGP), Heavy ion Collider (HIC).

INTRODUCTION

Experimental realization of super fluidity in cold atomic Fermi gas has given the study of BCS-BEC crossover a strong boost over the past decade[1-5]. Moreover the main attention has been paid to the strongly interacting limit, where the s-wave scattering length 'a' is large. In particular the unitary limit where the scattering length diverges has become a test point of theories[6-8].

In a cold Fermi gas with an equal mixture of two hyperfine states, wide-range control of the short-range inter atomic interaction via Feshbach resonance[9] has enabled the mapping of a new landscape of super fluidity popularly known as BCS-BEC crossover super fluidity. It connects BCS super fluids of loosely bound pairs to a BEC of tightly bound bosonic dimmers made up of two fermions of opposite sign. For a weakly attractive interaction $(\frac{1}{k_F a_s} \ll -1)$ with K_F being the Fermi momentum of a Free Fermi gas of same density, the gas shows a super fluid behaviour originating from many-body physics of Cooper pairs that are formed from weak pairing of atoms of opposite spin and momentum. For a strongly attractive interaction $(\frac{1}{k_F a_s} \gg 1)$ the gas shows a super fluid behaviour that can be explained by the Bose-Einstein condensation of tightly bound bosons made up of two fermions of opposite spins. In the crossover region/ unitary region $(\frac{1}{k_F a_s} \leq 1)$ bridging the two well-understood regions above, the gas is strongly interacting, which means that the interaction energy is comparable to the Fermi energy of a free Fermi gas of the same density. This also obeys perturbation many-body techniques. The physics in the so called unitary limit $|a_s| \to \infty$ or $|a_s| \to 0$ is very important in the sense that the thermodynamic properties are expected to be independent of the scattering length. Here the remaining length $1/K_F$ is approximately the inter particle distance appears in the equation of states while the interaction effect appears as a universal dimensionless parameter. The physics is unique in the sense that the constituents are independent and is non-perturbative due to strong interaction. The unitary Fermi gas has been the test bed for various theoretical techniques developed so far[10-15].

Cold atomic gases also provide insights into other forms of matter. Inside neutron stars, there might be quark matter made up of different ratio of quarks, and the imbalance might result in a new super fluid. Such super fluid is in Fulde-Farrel-Larkin-Ovehnnikov (FFLO) phase. Such phase has spatially-non uniform order parameter[16]. There is possibility that the electric charge neutrality of atoms might change their picture. The more information one can generate from the investigations on imblanced atomic Fermi gas[17]. Another example is the quark-gluon plasma (QGP) created in relativistic heavy-ion collider (RHIC)[18]. A QGP with a temperature of about 10^{12}K was produced by smashing nuclei together.

Measurements on its expansion after its creation show that the QGP is nearly perfect fluid with very small shear viscosity. A cloud of atomic Fermi gases at unitary shows the same strange behaviour while the origin of the similarity between hot and cold matter are to be speculated[19].

In this paper, we have studied the behaviour across resonance in the unitary limit $|a| \to \infty$. As one knows that in the unitary limit, the thermodynamic properties are expected to be independent of the scattering length and the only available length scale is the inter particle distance[20]. However this argument fails in the case of resonance when the scattering phase shift significantly changes over the energy range of the Fermi energy[21]. The second length scale called effective range plays an important role. Using the theoretical formalism of S.De Palo *et al.*[22], we have theoretically studied super fluidity of an atomic Fermi gas near the unitary limit. Using the theoretical formalism of Q. Chen[23], we have theoretically studied super fluid transition temperature in an atomic Fermi gas on 3D isotropic lattice with an attractive on-site interaction in the unitary limit. Our theoretically evaluated results are in good agreement with the works of other theoretical workers[24,25].

Mathematical formula used in the evaluation: One develops a model that is able to interpolate between two limits of a broad resonance. Here, the interaction is studied with the help of scattering length from low energy renormalization and of a narrow resonance in a broad Fermi sea. One models minimal interaction potential that is able to reproduce and independently tune the three relevant features of a Feshbach resonance. These are detuning and the width of the resonance and the back-ground scattering length a_{bg}. The thermodynamics properties of atomic gas in the unitary limit are investigated as function of resonance width by resorting to a mean-field solution of the BCS-like equations.

Mathematical formula used in the evaluation: One models the presence of Feshbach resonance by the interaction potential given by

$$V(r) = V_0 \quad r<r_0$$
$$= V_1 \quad r_0<r<r_1$$
$$= 0 \quad \text{otherwise} \qquad (1)$$

The potential is characterized by an attractive well depth V_0 and width r_0 and a barrier with height V_1 and width $r_1 - r_0 = r_w$. This barrier –well model allows one to incorporate the essential energy dependence of the scattering physics into a single channel scattering scheme. The well can support resonant states with a width dependent on the tunnelling through the potential barrier. Here, there are three distinct parameters like diluteness nr_0^3, the interaction strength na^3 and the width Δv of the resonance on the scale of the Fermi energy $E_F = \dfrac{\hbar^2 k^2{}_F}{2m}$ with $K_F = (3\pi^2 n)^{\frac{1}{3}}$. Δv can be expressed in terms of the matrix element g for the coupling between the closed and open channels as $\Delta v = g\sqrt{n}$, n is the particle density.

One requires that the diluteness conditions $na^3 <<1$ is always satisfied while staying within the unitary limit $na^3>>1$. One thus vary the width of the resonance by tuning the parameter $\frac{\Delta v}{E_F}$. The parameters of the model potential can in principle be adjusted to reproduce the scattering properties of an atomic sample[22]. One chooses two sets of parameters that are suitable for the physical behaviour of the sample. One fixes the scattering length to a large and positive values $a=5000a_0$ (a_0 is Bohr radius). This is for staying on BEC side of the resonance under unitary-limited conditions. The other parameter is range r_0 which is taken to be equal to $r_0 =500a_0$ such that $na^3=0.002<<1$. Now, one solves the scattering problem for V(r) shown in equation (1) in the standard way. Then, one determines the two-body scattering functions $\psi(r)$ and the T-matrix[26].

The T-matrix contains all the necessary information. The scattering length a is determined from its definition

$$\mathrm{Lim}_{k\to 0}T(k)=\frac{4\pi\hbar^2 a}{m} \tag{2}$$

The usual relation is

$$T(k)=2\pi\hbar^2 i[S(K)-1]/mk \tag{3}$$

The Feshbach form for the S-matrix in the one resonance parameterization is given by

$$S(k)=e^{-2ika_{bg}}[1-\frac{2ik|g|^2}{-\frac{4\pi\hbar^2}{m}(v-\frac{\hbar^2k^2}{m})+ik|g|^2}] \tag{4}$$

The parameter g is defined in terms of the effective range[27]

$$R_{eff}=-(\frac{4\pi\hbar^2}{m})[\frac{d^2T(K)^{-1}}{dk^2}]_{k=0} \tag{5}$$

R_{eff} is always negative close to resonance. One obtains

$$|g|^2=-\frac{8\pi\hbar^2}{mR_{eff}} \tag{6}$$

One has removed the background $a_{bg}=0$

Now, one finds the equilibrium state of the Fermi gas interacting via the non-local potential interaction modeled by V(r). The ground state is determined within the usual variational BCS scheme that utilizes the wave functions[28]

$$\left|\Phi_0\right\rangle = \Pi_k (u_k + v_k a^{\dagger}{}_{k\uparrow} a^{\dagger}{}_{-k\downarrow}) \left|0\right\rangle \tag{7}$$

where $a^{\dagger}{}_{k\sigma}$ is the creation operator for the electrons in spin σ. Φ_0 is normalized giving the condition $\left|u_k\right|^2 + \left|v_k\right|^2 = 1$. The expected value of the ground state energy at T=0 is given by

$$E_0 = \Sigma_k 2\varepsilon_k \left|v_k\right|^2 + \Sigma_{kk'} V_{kk'} u_k v^*{}_k u_{k'} v^*{}_{k'} + \Sigma_{kk'} V_{kk'} u_k u^*{}_k v_{k'} v^*{}_{k'} \tag{8}$$

Now, one solves the above equation by converting the summations into one-dimensional integral

$$\Sigma_k V_{kk'} F(\vec{k}') = \frac{1}{(2\pi)^3} \int dq q^2 V(k,q) F(q) \tag{8}$$

V(k,q) is determined from the three-dimensional Fourier transform

$$V(k,q) = \frac{2\pi}{kq}(V_0 + V_1)\left[\frac{\sin r_0 \left|k+q\right|}{\left|k+q\right|} - \frac{\sin r_0 \left|k-q\right|}{\left|k-q\right|}\right]$$

$$- V_1\left[\frac{\sin r_1 \left|k+q\right|}{\left|k+q\right|} - \frac{\sin r_1 \left|k-q\right|}{\left|k-q\right|}\right] \tag{9}$$

The BCS solution results from the minimization of the free energy with respect to the variational parameter u_k and v_k and chemical potential μ fixed by the constraint of particle-density conservation

$$f = \left\langle\Phi_0\left|H\right|\Phi_0\right\rangle - \mu\left\langle\Phi_0\left|N\right|\Phi_0\right\rangle \tag{10}$$

The two resulting self-consistent equations for the isotropic super fluid gap and the particle density are given by

$$\Delta(k) = \frac{1}{(2\pi)^3} \int dq q^2 V(k,q) \frac{\Delta(q)}{2E(q)} \tag{11a}$$

$$n = \frac{1}{(2\pi)^3} \int d\vec{k}\, n_k \tag{11(b)}$$

$$n_k = 1 - \frac{\xi(\vec{k})}{E(\vec{k})} \tag{11(c)}$$

In equations 11(a) to 11(c) the excitation energy E(k) is expressed as

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$$E(k) = \sqrt{\Delta(k)^2 + \xi(k)^2} \tag{12}$$

Here, $\Delta(k)$ is gap function and $\xi(k^{\rightarrow})$ is the single-particle energy and is given by

$$\xi(k^{\rightarrow}) = \varepsilon_k - \mu + \frac{1}{2} V_{k=q=0} n - \Sigma_k V_{kk'} n_{k'} \tag{13}$$

In this equation, the terms second and third contains the Hartee-Fock corrections to the single particle energy. Equations 11(a) and 11(b) are solved self-consistently to find $\Delta)k)$ at each k-vector and μ .The Bogoliubov parameters u_k and v_k were evaluated and the values are given by

$$u_k = \Delta(k)[\frac{1}{2}(1 + \frac{\xi(k)}{E(k)})]^{\frac{1}{2}} \tag{14(a)}$$

$$v_k = [\frac{1}{2}(1 - \frac{\xi(k)}{E(k)})]^{\frac{1}{2}} \tag{14(b)}$$

At finite temperature, the equations are modified to include Fermi function f**(k)** which is given by

$$f(k^{\rightarrow}) = [\exp((\xi(k^{\rightarrow}) / K_\beta T)]^{-1} \tag{15(a)}$$

$$\Delta(k) = \frac{1}{(2\pi)^3} \int dqq^2 V(k,q) \frac{\Delta(q)}{E(q)} (1 - 2f(q)) \tag{15(b)}$$

$$n = \frac{1}{(2\pi)^3} \int dk^{\rightarrow} n_k \tag{15(c)}$$

$$n_k = 2{v_k}^2 (1 - f(k) + 2{u_k}^2 f(k)) \tag{15(d)}$$

The single-particle excitation energy $\xi(k^{\rightarrow})$ is modified accordingly by substituting the new n_k from equation 15(c) and 15(d).

Evaluation of super fluid transition temperature of atomic Fermi gas on an optical lattice in an unitary limit: Now, one studies the super fluid transition temperature of atomic Fermi gas on an optical lattice in an unitary limit. In an unitary limit, scattering length diverges. On a lattice, the fermion dispersion is written as[28]

$$E_{k^{\rightarrow}} = 2t(3 - \cos k_x - \cos k_y - \cos k_z) - \mu$$

$$= \varepsilon_{\vec{k}} - \mu \tag{16(a)}$$

Here, t is nearest neighbour hopping integral, $\varepsilon_{\vec{k}}$ is the kinetic energy and μ is the chemical potential. One uses Lippman-Schwinger relation which is written as[29]

$$\frac{m}{4\pi a\hbar^2} = \frac{1}{V} + \Sigma_k \frac{1}{\varepsilon_{\vec{k}}} \tag{16(b)}$$

Critical coupling strength is given by

$$U_c = -\frac{1}{\Sigma_k \frac{1}{\varepsilon_{\vec{k}}}} \tag{16(c)}$$

Fermion self-energy comes from two contributions (i) super fluid condensate (ii) finite momentum pairs. Fermion self-energy is given by

$$\Sigma_k = \Sigma_{sc}(k) + \Sigma_{pg}(k) \tag{16(d)}$$

$$\Sigma_{sc}(k) = -\Delta^2{}_{sc} G_0(\vec{k}) \tag{16(e)}$$

$$\Sigma_{pg}(k) = \Sigma_Q t_{pg}(Q) G_0(Q-K) \tag{16(f)}$$

Δ_{sc} is the super fluid order parameter and $\Sigma_{sc}(k)$ vanishes at and above T_C. The finite momentum T-matrix is given by

$$t_{pg}(Q) = \frac{U}{1 + U\chi(Q)} \tag{17}$$

Here U is on-site attractive interaction, t_{pg} is calculated from summation of ladder diagrams in the particle-particle channels and Q is the pair momentum. Pair susceptibility $\chi(Q)$ is given by

$$\chi(Q) = \Sigma_k G(K) G(Q-K) \tag{18(a)}$$

K is in four vector notation

$$K = (i\omega_l, \vec{k}) \tag{18(b)}$$

$$Q = (i\Omega_n \vec{q}) \tag{18(c)}$$

$$\Sigma_K = T\Sigma_l \Sigma_n \tag{18(d)}$$

$$\Sigma_Q = T\Sigma_n \Sigma_q \qquad 18(e)$$

Here $\omega_l(\Omega_n)$ are the odd (even) Matsubara frequencies. The pseudo gap parameter Δ_{pg} is given by

$$\Delta_{pg} = -\Sigma_Q t_{pg}(Q) = Z^{-1}\Sigma_q b(\Omega_q) \qquad (19)$$

Now, one solves the gap equation

$$1 + U\Sigma_k \frac{(1-2f(E_k))}{2E_k} = 0 \qquad (20)$$

Here, b(x) is Bose distribution function and f(x) is Fermi-Dirac distribution function. Number equation n is given by

$$n = \Sigma_k [1 - \frac{\xi_k}{E_k}(1-2f(E_k)] \qquad (21)$$

Now, equations (19), (20) and (21) form a closed set. From a given interaction U, These equations are solved self-consistently to calculate T_C, Δ and μ respectively

DISCUSSION OF RESULT

Using the theoretical formalism of S.De Palo etal[22], we have studied super fluidity of an atomic Fermi gas near the unitary limit. The studies show that the thermodynamic properties of an atomic gas near the unitary limit are not universal as long as the resonance is narrow on the scale of Fermi energy. This formalism uses a mean-field BCS-like approach with a model potential that is able to reproduce the main character of Feshbach resonance. The formalism also gives a complete theoretical description of the crossover from BEC of composite bosons to BCS-type of Cooper pairs. It was also observed that the thermodynamic properties of the super fluid with large and positive scattering length on BEC side of resonance are affected by the energy dependence of the phase shift. The narrow resonance favours the occurrence of the strong coupling super fluidity. **In table T1,** we have shown the evaluated results of gap function $\Delta(k)$ as a function of $\frac{k}{k_F}$ for different values of $g\sqrt{n}/E_F$ in the unitary limit. We have taken the values of $g\sqrt{n}/E_F$=0.98, 1.2, 1.4, 1.6 and 8.0. We observed that the gap function show the damped oscillatory behaviour in the scale of $1/r_0$. This is the manifestation of the pairing potential. The wavelength and damping coefficient of oscillations are almost independent of the resonance width. Our theoretically evaluated results show that $\frac{\Delta(k)}{E_F}$ is very sensitive with $\frac{k}{k_F}$. **In table T2**, we have presented an evaluated results of $\frac{\Delta_0}{E_F}$ as a function of $g\sqrt{n}/E_F$. One observes that there is a BCS-type of solution where resonance shrinks on the scale of E_F. The ratio $\frac{\Delta_0}{E_F}$ is seen to decrease as a function of $g\sqrt{n}/E_F$ and becomes almost constant at values $g\sqrt{n}/E_F$=8.0. We have kept different parameters like scattering length a=5000a_0 and $nr_0^3=2x10^{-3}$.

Table T1: An evaluated result of the gap function $\frac{\Delta(k)}{E_F}$ as a function of $\frac{k}{k_F}$ for different values of $g\sqrt{n}/E_F$=0.98, 1.2, 1.4, 1.6 and 6.0. in the unitary limit. The other parameters are scattering length a= $5000a_0$, $nr^3{}_0 = 2x10^{-3}$ as a function of resonance width

$\frac{k}{k_F}$	⟵ $\frac{\Delta(k)}{E_F}$ ⟶				
	$g\sqrt{n}/E_F$ =0.98	$g\sqrt{n}/E_F$ =1.2	$g\sqrt{n}/E_F$ =1.4	$g\sqrt{n}/E_F$ =1.6	$g\sqrt{n}/E_F$ =6.0
0.0	12.6	10.5	8.44	6.53	5.64
5.0	5.43	4.23	3.78	2.87	1.78
10.0	-2.42	-3.27	-4.64	-5.82	-6.22
15.0	-1.65	-1.46	-2.51	-3.27	-4.07
20.0	-1.04	-0.94	-1.32	-1.88	-2.26
25.0	2.82	1.87	1.08	0.59	5.42
30.0	3.50	2.22	1.88	2.86	3.32
35.0	0.86	0.73	0.63	1.07	1.57
40.0	-1.80	-2.64	-1.45	-0.84	0.53
45.0	-0.25	-1.25	-1.03	-1.32	-0.89
50.0	1.86	0.47	0.52	0.47	0.15
55.0	0.52	0.32	0.40	0.23	0.07

Table T2: An evaluated result of $\frac{\Delta(0)}{E_F}$ as a function of $g\sqrt{n}/E_F$ in the unitary limit $\Delta(0) = \Delta(k=0)$, keeping $a = +5000a_0$ and $nr^3{}_0 = 2x10^{-3}$

$g\sqrt{n}/E_F$	⟵ $\frac{\Delta(0)}{E_F}$ ⟶
0.0	25.6

1.00	18.45
2.00	12.80
3.00	6.79
4.00	4.86
5.00	3.58
6.00	2.37
7.00	1.76
8.00	1.23
9.00	0.98
10.00	0.76
15.00	0.57

In table T3, we have presented an evaluated results of chemical potential $\frac{\mu}{E_F}$ as a function of $g\sqrt{n}/E_F$. Others parameters are same as in **table T2.** Our theoretical results show that chemical potential is large and negative and increases with $g\sqrt{n}/E_F$ and for $g\sqrt{n}/E_F$ =10, it becomes positive. We also observed that the ground state energy equation (8) behaviour that narrowing the resonance has now effect of increasing the limit of the interaction. These results are obtained on the BEC side of the resonance. It also suggests that high quality resonance leads to a non universal regime in the unitary limit.

Table T3: An evaluated result of chemical potential $\frac{\mu}{E_F}$ as a function of $g\sqrt{n}/E_F$ in the unitary limit keeping $a = +5000a_0$ and $nr^3{}_0 = 2x10^{-3}$

$g\sqrt{n}/E_F$	$\longleftarrow \frac{\mu}{E_F} \longrightarrow$
0.0	-14.67
1.00	-13.69
2.00	-12.39
3.00	-11.6
4.00	-6.23
5.00	-5.65
6.00	-4.89
7.00	-3.27
8.00	-1.64
9.00	-0.85
10.00	0.26
12.00	0.79

15.00	1.26
20.00	2.47

In table T4, we have shown the evaluated results of $\frac{\Delta_0}{k_\beta T_F}$ as a function of $\frac{T}{T_F}$ for different values of $g\sqrt{n}/E_F$ in the unitary limit. The others parameters are same as in **table T2 and T3**. Our theoretical results show that the ratio $\frac{\Delta_0}{k_\beta T_F}$ decrease with $\frac{T}{T_F}$ for all values of $g\sqrt{n}/E_F$. This result also indicates as to how narrow resonance favours the emergence of super fluid state with regard to both the gap strength and the value of the transition temperature. The smooth temperature behaviour of Δ_0 near the transition is a manifestation of the short ranged, bosonic character of the resulting state. Using the theoretical formalism of Q. Chen[23], we have theoretically studied super fluid transition temperature in an atomic Fermi gas on 3D isotropic lattice with an attractive on-site interaction in the unitary limit.

Table T4: An evaluated result of $\frac{\Delta(0)}{K_F T_F}$ as a function of $\frac{T}{T_F}$ for different values of $g\sqrt{n}/E_F$ =0.95, 1.10, 1.20, 1.30 and 3.00 in the unitary limit keeping $a = +5000a_0$ and $nr^3_0 = 2x10^{-3}$

$\frac{T}{T_F}$	← $\frac{\Delta(0)}{K_F T_F}$ →				
	$g\sqrt{n}/E_F$ =0.95	$g\sqrt{n}/E_F$ =1.10	$g\sqrt{n}/E_F$ =1.20	$g\sqrt{n}/E_F$ =1.30	$g\sqrt{n}/E_F$ =3.00
0.00	2.25	2.13	2.00	1.92	1.85
0.10	2.12	1.88	1.78	1.65	1.78
0.20	1.89	1.76	1.65	1.48	1.67
0.30	1.65	1.57	1.49	1.37	1.42
0.40	1.39	1.40	1.38	1.30	1.33
0.50	1.28	1.29	1.30	1.24	1.22
0.60	1.22	1.23	1.24	1.21	1.19
0.70	1.19	1.18	1.20	1.15	1.14
0.80	1.17	1.16	1.16	1.10	1.00
0.90	1.12	1.13	1.14	0.92	0.87
1.00	0.98	1.00	0.98	0.76	0.72
1.10	0.76	0.89	0.72	0.66	0.64
1.20	0.58	0.67	0.67	0.51	0.53
1.30	0.35	0.42	0.38	0.34	0.33

1.40	0.28	0.29	0.25	0.21	0.20
1.50	0.17	0.19	0.18	0.16	0.14

In table T5, we have shown an evaluated result of $\frac{T_C}{6t}$ as a function of on-site interaction $\frac{-U}{6t}$ for various densities n. Here, t is the nearest neighbour hopping integral, U is on-site attractive interaction. The evaluation is performed for various densities n=0.8, 0.6, 0.2, 0.05 and 0.005. Our theoretically evaluated results indicate that $\frac{T_C}{6t}$ increase attain maximum value and then decrease as a function of $\frac{-U}{6t}$ for various densities. Here, 6t is half band width. For n=0.8, the maximum T_C occurs on the BEC side of unitary. Even at n=0.005, the maximum is on BCS side.

Table T5: An evaluated result of $\frac{T_C}{6t}$ as a function of attractive on-site interaction $-\frac{U}{6t}$ for various densities n, the evaluation is performed on a 3D isotropic lattice in the unitary limit.

$-\frac{U}{6t}$	$\longleftarrow \frac{T_C}{6t} \longrightarrow$				
	n=0.8	n=0.6	n=0.2	n=0.05	n=0.005
0.00	0.002	0.004	0.006	0.008	0.010
0.50	0.014	0.024	0.034	0.042	0.055
1.00	0.022	0.048	0.051	0.059	0.067
1.50	0.172	0.125	0.105	0.095	0.089
2.00	0.154	0.097	0.082	0.076	0.074
2.50	0.132	0.084	0.077	0.062	0.061
3.00	0.104	0.065	0.063	0.058	0.053
3.50	0.095	0.058	0.054	0.049	0.045
4.00	0.076	0.047	0.038	0.033	0.029
4.50	0.067	0.032	0.027	0.024	0.018
5.00	0.054	0.023	0.018	0.016	0.012
5.50	0.047	0.010	0.009	0.008	0.006
6.00	0.008	0.006	0.004	0.002	0.0015

In table T6, we have shown an evaluated result of $\frac{T_C}{E_F}$ as a function of $\frac{-U}{6t}$ for various densities n. Our theoretically obtained results indicate that as n increases the peak becomes narrower and narrower and moves closer to unitary. Here, T_C follows a functional form

$$T_C \alpha(-\frac{t^2}{U})$$

This is due to virtual ionization during the pair hopping in the BEC regime[30,31]. **In table T7,** we have presented an evaluated result of $\frac{T_C}{E_F}$ as a function of $n^{\frac{1}{3}}$ on a3D isotropic lattice in the unitary limit. We observed for low n, the lattice effect is expected to vary as $n^{1/3}$ to the leading order

$$\frac{T_C(n)}{E_F(n)} = \frac{T_c(0)}{E_F(0)} - \alpha a_0 n^{\frac{1}{3}} + \alpha(a^2{}_0 / n^{\frac{2}{3}}) + --$$

where α is proportionality constant. Here, $a_0 n^{\frac{1}{3}}$ represents the ratio between the lattice period and the mean inter particle distance. Our theoretically evaluated results show that $\frac{T_C}{E_F}$ decrease as $n^{\frac{1}{3}}$ and attain minimum value at $n^{\frac{1}{3}}$ =0.45 and after that it increase sharply. There is some recent calculations[32-38] which also reveals the similar behaviour.

Table T6: An evaluated result of $\frac{T_C}{E_F}$ as a function of $-\frac{U}{6t}$ for various densities n=9.2, 0.05, 0.01, 0.005 and 0.001 for 3D isotropic lattice in the unitary limit

$-\frac{U}{6t}$	$\longleftarrow \frac{T_C}{E_F} \longrightarrow$				
	n =0.2	n =0.05	n =0.01	n =0.005	n =0.001
0.00	0.00	0.00	0.00	0.00	0.00
0.20	0.04	0.05	0.07	0.06	0.08
0.40	0.06	0.07	0.09	0.09	0.12
0.60	0.08	0.11	0.11	0.12	0.17
0.80	0.12	0.13	0.13	0.14	0.25
1.00	0.17	0.25	0.27	0.26	0.28
1.15	0.26	0.24	0.23	0.22	0.23
2.00	0.20	0.22	0.20	0.21	0.21
2.20	0.18	0.20	0.18	0.17	0.19

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2.50	0.16	0.18	0.16	0.15	0.17
3.00	0.14	0.16	0.15	0.14	0.16
3.20	0.12	0.15	0.13	0.12	0.15
3.50	0.10	0.14	0.11	0.10	0.13
4.00	0.09	0.12	0.09	0.08	0.11
4.20	0.08	0.10	0.08	0.07	0.09
4.50	0.07	0.09	0.07	0.06	0.05
5.00	0.06	0.08	0.06	0.05	0.04

Table T7: An evaluated result of T_C/E_F as a function of $n^{1/3}$ on a 3D isotropic optical lattice in the unitary limit

$n^{1/3}$	T_C/E_F
0.00	0.26
0.10	0.22
0.15	0.20
0.20	0.18
0.25	0.17
0.30	0.16
0.35	0.15
0.40	0.14
0.45	0.13
0.50	0.16
0.55	0.19
0.60	0.22
0.65	0.24
0.70	0.30
0.75	0.32
0.80	0.34
0.85	0.36
0.90	0.38
1.00	0.43

CONCLUSION

From the above theoretical investigations and analysis, we have come across the following conclusions:

1. We have studied super fluidity of an atomic Femi gas near the unitary limit. Our evaluated results of gap function $\Delta(k)$ as a function of $\frac{k}{k_F}$ for different values of $g\sqrt{n}/E_F$ show an oscillatory damped behaviour. This is because of manifestation of the pairing potential.

2. Our theoretically evaluated values of $\frac{\Delta_0}{E_F}$ and $\frac{\mu}{E_F}$ as a function of $g\sqrt{n}/E_F$ show a BCS-like solution. Our results of $\frac{\Delta_0}{E_F}$ decrease and $\frac{\mu}{E_F}$ increase with $g\sqrt{n}/E_F$. Here, the chemical potential is large and negative. The results are obtained on BEC side of resonance. This is the nontrivial regime in the unitary limit.

3. Our theoretically evaluated result of $\frac{\Delta_0}{k_\beta T_F}$ as function of $\frac{T}{T_F}$ for different values of $g\sqrt{n}/E_F$ indicate that $\frac{\Delta_0}{k_\beta T_F}$ decrease with $\frac{T}{T_F}$ for all values of $g\sqrt{n}/E_F$. These results also show that the narrow resonance favours the emergence of the super fluid state both in regard to gap strength and the value of transition temperature. This behaviour is an indication of short ranged, bosonic character of the resulting state.

4. Our theoretically evaluated results of $\frac{T_C}{6t}$ as a function of $\frac{-U}{6t}$ for different values of n indicate that the maximum T_C occurs on BEC side of unitary. Then it moves to BCS side as n decrease. Even at density as low as n=0.005, the maximum is still on BCS side.

5. Our theoretically evaluated results of $\frac{T_C}{E_F}$ as a function of $\frac{-U}{6t}$ for different values of n show that as n decrease the peak becomes narrower and narrower and moves closer to unitary. Beyond the unitary limit the value of n=0.005, $\frac{T_C}{E_F}$ exhibit a rapid falloff with pairing potential. The decreasing trend follow a functional form of T_C as $T_C \alpha (-t^2/U)$.

6. Our theoretically evaluated results of $\frac{T_C}{E_F}$ as a function of $n^{\frac{1}{3}}$ show that for low n, the lattice effect is expected to vary as $n^{\frac{1}{3}}$. We also observed that $\frac{T_C}{E_F}$ is highly nonlinear as a function of $n^{\frac{1}{3}}$. As n decreases $\frac{T_C}{E_F}$ decreases and attains minimum value at $n^{\frac{1}{3}}$ =0.45 and after that it increases sharply.

7. These theoretical investigations and analysis may be quite relevant for constructing a complete theoretical description of the crossover from BEC of composite bosons and BCS of Cooper pairs. Till today, we do not have very successful theory for high T_C –superconductors. These studies will provide good insight in this direction.

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